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stringed instruments

Calculating string guages and tensions

Back many years ago the Guild of American Luthiers published as Data Sheet 144, from Max Krimmel, the following formula:
$T = 4 \text{ x } F^2 \text{ x } L \text{ x } M / 980621$
where
T = Tension in Kgs
F = Frequency in Hz
L = Vibrating string length in cm
M = Mass of that length in gm
980621 is a constant which has something to do with gravity (don’t ask me I have no idea)

If you want to work out the gauges of strings for a predetermined scale length and tension the formula becomes:
$M = 980621 \text{ x } T / 4 \text{ x } F^2 \text{ x } L$

The tricky thing to know is what tension you want for any particular instrument. This is a matter of both trial & error and personal preference, but a good starting point are D’Addario string packets which now give the tension for each string for a standard guitar scale length at common pitches For example a light gauge guitar set has string tensions of 10-12 kgs per string, and that is probably a good starting point.

For bouzoukis I go a bit lighter, around 8-10 kgs per string. Even so there is a fair bit of variation across the set I use on a 660mm (26”) scale bouzouki. At that scale a .040” *G* string has 9.56kg tension, a .030” *D* has 12.64kg, a .017”plain steel *a* has 9.89kg and a .010” high *d* has only 6.13kg. Tuning that *d* up to *e* raises the tension to 7.72kg, while going to a .012” for the *d* has 9.2 kg. Nevertheless these strings give an even ‘feel’ across the strings, and that always has to be the critical factor

While these are the gauges that work on my instruments, other instruments are going to respond differently. For any stringed instrument, whatever the scale length or tuning there is a critical range of string gauges that will work. As an example, for a standard 25” or so acoustic guitar 1st string, tuned to e, various sets use strings from a .010” up to a .013”. Any lighter and there will not be the tension to drive the soundboard and much heavier is going to be really hard to play and could well cause damage to the instrument.

The other factor is the ‘tone’ or ‘colour’ of the sound, which is the result of the harmonics of the fundamental note. The shorter and thicker the string is for a given pitch, the less harmonics are being produced. An example here is the diference between a long scale bouzouki of 26” scale and a 22” octave mandolin. I use a .040” for the G on the long scale and a .046” on the short scale and the difference in sound is quite marked. At the extreme end of this are the octave mandolas commonly used in European mandolin orchestras, tuned GDae with great fat strings for all of them and a sound best described as ‘plodge’.

Of course the information that is critical for applying this formula is the string mass, so over a period of a few weeks every time I restrung an instrument I cut off 10cm from the end of the strings. The strings came from a number of manufacturers, and both plain steel and wound strings were accumulated. The wound strings had a variety of wrap compositions (brass, bronze, phosphor bronze), but neither this nor the manufacturer seemed to make much difference to the mass of the string.

The Powerhouse Museum in Sydney kindly let me weigh the bits of string in their lab, and these were plotted into graphs.

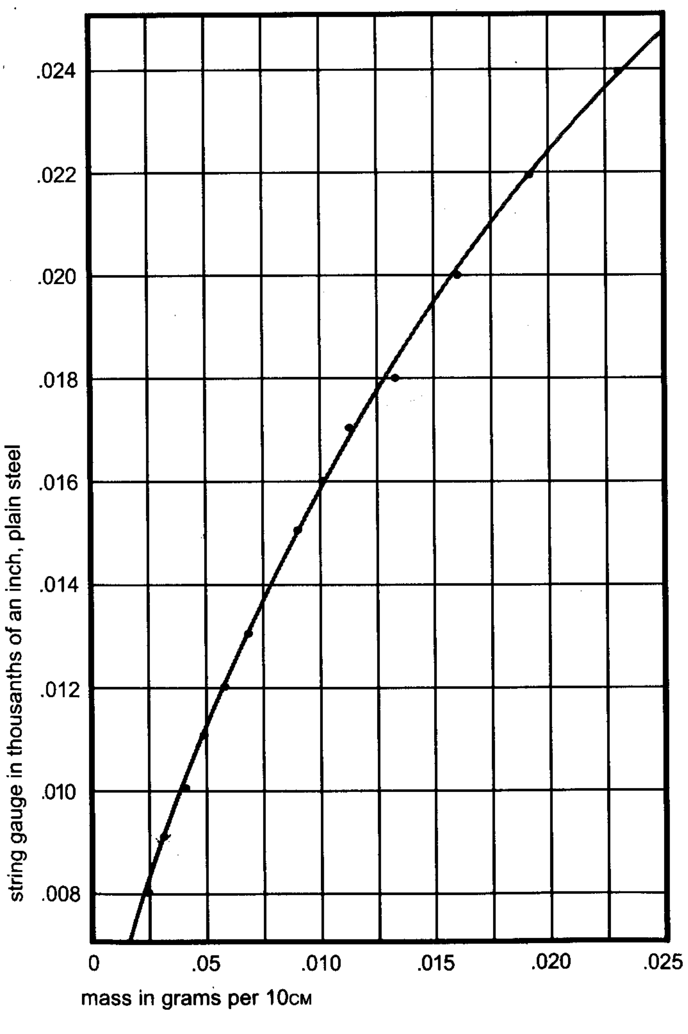
Plain steel strings

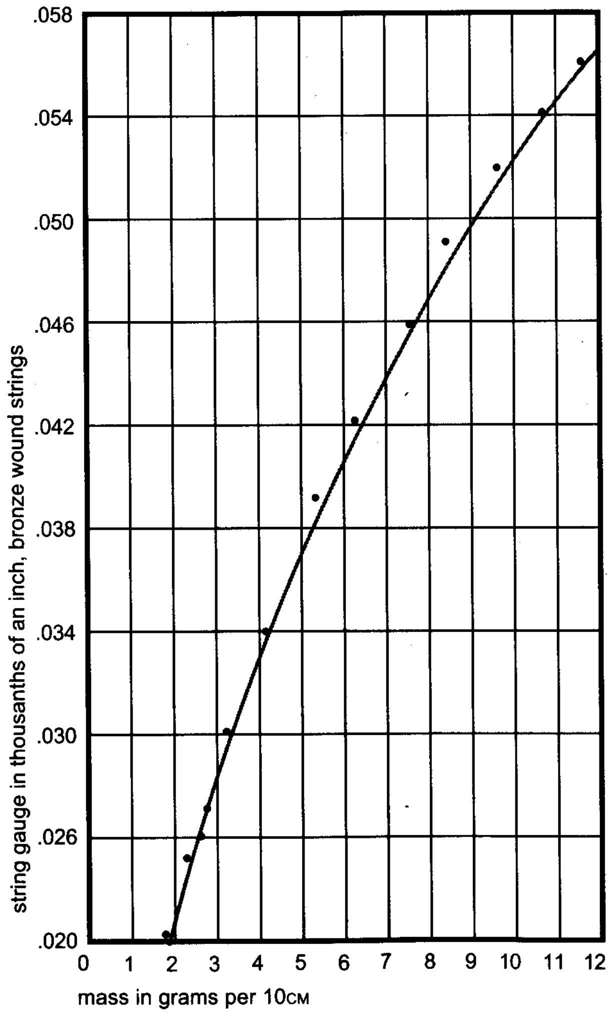
Gauge	Mass in gm per 10cm
.008"	.025
.009	.03
.010	.04
.011	.048
.012	.057
.013	.065
.015	.09
.016	.102
.017	.114
.018	.135
.020	.162
.022	.193
.024	.231

Wound Strings

Gauge	Mass in gm per 10cm
.020"	.17
.025	.225
.026	.25
.027	.275
.030	.325
.034	.25
.039	.525
.042	.625
.046	.75
.049	.835
.052	.96
.054	1.075
.056	1.16

The graphs show quite a smooth curve for both kinds of string, which makes reading information off them quite simple.





The other information required is a table of pitches and frequencies

E = 82.407 Hz (guitar 6th)
F = 87.307
F#= 92.499
G = 97.999
G#= 103.826
A = 110 (guitar 5th)
A#= 116.541
B = 123.471
C = 130.813
C#= 138.591
D = 146.832 (guitar 4th)
D#= 155.563

Here are a couple of examples of calulation that can be made:

Finding the tension of a .012" string tuned to E on a 63cm (24.8")scale instrument.

F = 329.6 Hz F^2 = 108636.16
L = 63
M = .057 x 6.3 = .359gm

T = 4 x 108636.16 x 63 x .359 / 980621

= 10.02 Kg or 22.05lb

Finding gauge of string required for 110Hz A (guitar fifth) at a tension of 7kg for a 66cm scale

T = 7 Kg
F = 110 Hz F^2 = 12100
L = 66 cm

M = 980621 x 7 / 4 x 12100 x 66

= 2.15gm divide this by 6.6
= 3.3gm and from the graph the string required is a.030" wound string.

For the metricly challenged 1kg = 2.2 lb and 1" = 25.4mm

This was originally published in American Lutherie No 2 June 1985. The Guild of American Luthiers <www.luth.org> is a fine organization and their quarterly journal is essential reading for any stringed instrument builder.

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